Application of Enhanced Hidden Markov Model in Stock Price Prediction

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ABSTRACT

The main focus of this research is the enhancement of the Hidden Markov Model by using some features of Neural Networks and the forecasted values of predictors by Seasonal Autoregressive Integrated Moving Average. The enhanced method was used to predict the close price of stocks whose predictors are open price, high price, low price, and volume of Apple and Nokia data. The performance of the method was measured using the Mean Absolute Percentage Error of the predicted price. The result was compared against the actual close price by using the paired T-test. The testing of the hypothesis showed that the Enhanced Hidden Markov Model obtained more than 94% accuracy rate. It also shows that in Apple data, the predicted close price of the Enhanced Hidden Markov Model is significantly better than the predicted close price of Neural Networks. Using Nokia data, the test claims that there is no difference between the performance of Enhanced Hidden Markov Model and Neural Network in prediction.

Keywords: Stock Price Prediction; Enhanced Hidden Markov Model; Neural Networks.

1 Introduction

Forecasting and prediction is now a current trend in the field of research. This is to optimize the resources or data available due to the advancement of technology. Various fields such as Computer Sciences, Medicine, Marketing, Economics and Business employed this type of researchers since most companies aimed to create an insight into their available data. In order to properly treat the data on hand, these companies spend a lot of resources. This is because they are fully aware that the extracted knowledge from the available data can be turned into the best decisions and solutions.

In the field of business, especially the stock market, hiring a good adviser or someone who will check the stock prices and trends is just an ordinary scenario. This is to study the chaotic system containing a large number of data and various factors that contribute to the price of the stock. A lot of researchers presented their study regarding the stock market prediction which is limited only in the index status. However, this paper created a forecast and prediction of prices specifically the close price which is more complicated because the predicted results are dependent on different variables or predictors.

Seasonal Autoregressive Integrated Moving Average (SARIMA) is one of the known techniques in forecasting in time series data with seasonality. SARIMA is a good model for prediction since its overall accuracy rate is more than 80% [1], [6], [11], [12], [15]. To improve the forecast and prediction, this paper used only the SARIMA in forecasting the predictors such as open price, low price, high price and volume. After employing the SARIMA in the forecasting of predictors, the Neural Networks were used to determine the predicted stock price known as Close price. Most of the papers mentioned above use the SARIMA
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for forecasting the future values of the stocks without incorporating into another model especially the prediction models used in this study.

Neural Networks is one of the famous methods in predicting patterns when the relationship between inputs and outputs were complex. It has a robust ability to discover the relationship in the input data set without a priori assumption of the knowledge of the relation between the input and the output data and it can be used to build a model that identifies unknown hidden patterns in data which can be further used for prediction purposes [2], [10], [16]. The proposed method will be the combination of Seasonal Autoregressive Integrated Moving Average, Neural Networks and Hidden Markov Model. The developed model was presented in Section 5 of this paper and the model was called Enhanced Hidden Markov Model.

2 Seasonal Autoregressive Integrated Moving Average

Predicting a time series data is a challenging task [5]. In this paper, the training data underwent the forecasting using Seasonal Autoregressive Integrated Moving Average. The parameters \((p, d, q) = (2, 1, 2)\) and \((P, D, Q) = (0, 1, 1)\) for Apple data and \((p, d, q) = (0, 2, 1)\) and \((P, D, Q) = (0, 1, 1)\) for Nokia data where \(p\) is the number of autoregressive terms, \(d\) is the number of nonseasonal differences needed for stationarity, \(q\) is the number of lagged forecast errors in the prediction equation, \(P\) is the number of seasonal autoregressive terms, \(D\) is the number of seasonal differences and \(Q\) is the number of seasonal moving average [3],[7], [17].

After the simulation of the test, the summarized derived forecasting model for Apple and Nokia are the following:

\[
\hat{Y}_t \text{(Apple)} = \mu + Y_t - 2 \phi_1 Y_{t-1} + \phi_2 Y_{t-2} - \theta_1 \epsilon_{t-1} - \Theta_1 \epsilon_t + \theta_1 \Theta_1 \epsilon_{t-1}
\]

\[
\hat{Y}_t \text{(Nokia)} = Y_t - 2 \phi_1 Y_{t-1} + \phi_2 Y_{t-2} - \theta_1 \epsilon_{t-1} - \Theta_1 \epsilon_t + \theta_1 \Theta_1 \epsilon_{t-1}
\]

where

\(\hat{Y}_t\) = forecasted close price

\(\mu\) = intercept

\(\phi\) = coefficient of autoregressive

\(\theta\) = coefficient of moving average

\(\Theta\) = coefficient of seasonal moving average

Based on the equation of Apple as presented in (1), the derived equations for the predictors of Apple close price was shown in (3), (4), (5) and (6). The forecasting equation for Nokia Data was shown in (7), (8), (9) and (10) using (2).

\[
\hat{Y}_t \text{(Open Price)} = 0.0000465 + Y_t - 2 + (0.89215)Y_{t-1} + (0.01475)Y_{t-2} - (1.55053)\epsilon_{t-1} - (0.99581)\epsilon_t + (-0.59327)(0.99581)\epsilon_{t-1}
\]

(3)

\[
\hat{Y}_t \text{(High Price)} = Y_t - 2 (-0.39977)Y_{t-1} + (-0.55395)Y_{t-2} - (-0.50489)\epsilon_{t-1} - (0.99644)\epsilon_t + (-0.55573)(0.99644)\epsilon_{t-1}
\]

(4)

\[
\hat{Y}_t \text{(Low Price)} = Y_t - 2 (0.00619)Y_{t-1} + (-0.34648)Y_{t-2} - (-0.06651)\epsilon_{t-1} - (0.99808)\epsilon_t + (-0.28512)(0.99808)\epsilon_{t-1}
\]

(5)

\[
\hat{Y}_t \text{(Volume)} = Y_t - 2 (1.27347)Y_{t-1} + (-0.32416)Y_{t-2} - (1.78506)\epsilon_{t-1} - (0.99004)\epsilon_t + (-0.78722)(0.99004)\epsilon_{t-1}
\]

(6)

\[
\hat{Y}_t \text{(Open Price)} = -0.00076 + (0.10333)\epsilon_{t-1} - (0.99282)\epsilon_t + (-0.009067)(0.99282)\epsilon_{t-1}
\]

(7)
\[ Y_t \text{ (High Price) } = -(0.00152)e_{t-1} - (0.99528)e_t + (0.00836)(0.99528)e_{t-1} \] (8)

\[ Y_t \text{ (Low Price) } = -(0.001694)e_{t-1} - (0.99152)e_t + (0.07875)(0.99152)e_{t-1} \] (9)

\[ Y_t \text{ (Volume) } = -(0.6457)e_{t-1} - (0.99895)e_t \] (10)

The forecasted values of predictors for two weeks or ten days was shown in Table 1 and Table 2. The accuracy and error of the forecasting of predictors for both Apple and Nokia were presented in Table 3 which was computed using Mean Absolute Percentage Error (MAPE) as shown in (11).

\[ MAPE = 100 \times \frac{1}{n} \sum_{t=1}^{n} \left| \frac{A_t - P_t}{A_t} \right| \] (11)

where:

- \( A_t \) = the actual stock price
- \( P_t \) = forecasted/predicted stock price
- \( n \) = total number of test cases

**Table 1: Forecasted Values of Predictors Using Apple Data**

<table>
<thead>
<tr>
<th>DATE</th>
<th>OPEN PRICE</th>
<th>HIGH PRICE</th>
<th>LOW PRICE</th>
<th>VOLUME</th>
</tr>
</thead>
<tbody>
<tr>
<td>2018-12-31</td>
<td>153.380</td>
<td>158.553</td>
<td>154.343</td>
<td>43774812</td>
</tr>
<tr>
<td>2019-01-01</td>
<td>153.912</td>
<td>158.576</td>
<td>154.203</td>
<td>4752396</td>
</tr>
<tr>
<td>2019-01-02</td>
<td>153.898</td>
<td>158.458</td>
<td>154.356</td>
<td>45231132</td>
</tr>
<tr>
<td>2019-01-03</td>
<td>153.672</td>
<td>158.478</td>
<td>154.346</td>
<td>45183497</td>
</tr>
<tr>
<td>2019-01-04</td>
<td>153.387</td>
<td>158.462</td>
<td>154.058</td>
<td>45681188</td>
</tr>
<tr>
<td>2019-01-07</td>
<td>152.448</td>
<td>157.989</td>
<td>153.846</td>
<td>45183497</td>
</tr>
<tr>
<td>2019-01-08</td>
<td>152.793</td>
<td>157.901</td>
<td>154.128</td>
<td>47494892</td>
</tr>
<tr>
<td>2019-01-10</td>
<td>153.128</td>
<td>158.117</td>
<td>154.168</td>
<td>46930942</td>
</tr>
<tr>
<td>2019-01-11</td>
<td>153.040</td>
<td>157.920</td>
<td>154.194</td>
<td>44601811</td>
</tr>
</tbody>
</table>

**Table 2: Forecasted Values of Predictors Using Nokia Data**

<table>
<thead>
<tr>
<th>DATE</th>
<th>OPEN PRICE</th>
<th>HIGH PRICE</th>
<th>LOW PRICE</th>
<th>VOLUME</th>
</tr>
</thead>
<tbody>
<tr>
<td>2018-12-31</td>
<td>5.6622</td>
<td>5.8203</td>
<td>5.6717</td>
<td>27380691</td>
</tr>
<tr>
<td>2019-01-01</td>
<td>5.6573</td>
<td>5.82</td>
<td>5.6538</td>
<td>29256318</td>
</tr>
<tr>
<td>2019-01-02</td>
<td>5.6618</td>
<td>5.8296</td>
<td>5.6495</td>
<td>28905019</td>
</tr>
<tr>
<td>2019-01-03</td>
<td>5.6615</td>
<td>5.8244</td>
<td>5.6378</td>
<td>31117286</td>
</tr>
<tr>
<td>2019-01-04</td>
<td>5.6608</td>
<td>5.8205</td>
<td>5.6426</td>
<td>28135446</td>
</tr>
<tr>
<td>2019-01-07</td>
<td>5.6506</td>
<td>5.8112</td>
<td>5.6463</td>
<td>27327953</td>
</tr>
<tr>
<td>2019-01-08</td>
<td>5.6455</td>
<td>5.8126</td>
<td>5.6431</td>
<td>29203580</td>
</tr>
<tr>
<td>2019-01-09</td>
<td>5.65</td>
<td>5.8222</td>
<td>5.6388</td>
<td>28852281</td>
</tr>
<tr>
<td>2019-01-10</td>
<td>5.6497</td>
<td>5.817</td>
<td>5.6271</td>
<td>31064548</td>
</tr>
<tr>
<td>2019-01-11</td>
<td>5.6489</td>
<td>5.8131</td>
<td>5.6319</td>
<td>28082708</td>
</tr>
</tbody>
</table>

**Table 3: Error and Accuracy of the Forecasted Predictors using Apple and Nokia Data**

<table>
<thead>
<tr>
<th>Predictors</th>
<th>Error (%)</th>
<th>Accuracy (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Apple</td>
<td>Nokia</td>
</tr>
<tr>
<td>Open</td>
<td>7.13487</td>
<td>2.07442</td>
</tr>
<tr>
<td>High</td>
<td>1.51459</td>
<td>1.86807</td>
</tr>
<tr>
<td>Low</td>
<td>1.66020</td>
<td>1.99107</td>
</tr>
</tbody>
</table>
3 Neural Networks

Neural Network is one of the best machine learning models for prediction and classification. It has a strong capability to determine the relationship in the input data set without a priori assumption of the knowledge of the relation between the input and the output data [10], [19]. It can be an aid to build a new model that identifies unknown hidden patterns in data sets [3], [8].

The role of Neural Networks in this study is to create a prediction of the stock price given that the open, low, high price and volume was treated as predictors.

The weights produced by the networks were implemented in the Enhanced Hidden Markov model. The model chart of the Neural Networks for both Apple and Nokia data was shown in Figure 1 and Figure 2. These results were implemented using XLStat.

![Figure 1: Model Chart of Neural Networks Using Apple Data](image1)

![Figure 2: Model Chart of Neural Networks Using Nokia Data](image2)

4 Hidden Markov Model

A Hidden Markov Model consists of two stochastic processes. The first stochastic process is the Markov chain which is characterized by states and transition probabilities wherein the states were hidden. On the other hand, the second stochastic process produces emissions observable at each moment, depending on a state-dependent probability distribution [4], [13], [18], [20].
The development of Hidden Markov Models is widely used in various studies all over the world. Just like the Markov Models and Markov Chain Models, Hidden Markov Models are widely used also in prediction and analysis. This method applied in both one (1) dimensional such as voice recognition and enhancement and two (2) dimensional images.

In this study, the Hidden Markov Model was explored and enhanced by modifying the prediction stage of the model where the result of Seasonal Autoregressive Integrated Moving Average and Neural Networks were used. The enhanced model was implemented to predict the future close price of the stock of Apple and Nokia.

5 Prediction using Enhanced Hidden Markov Model

Figure 3 shows the complete process flow of the development of Enhance Hidden Markov Model. The gathered data from www.finance.yahoo.com of Apple and Nokia from 12-29-2014 to 01-11-2019 was converted into Time Series Data. The missing data were treated by getting the average price or volume per week. After the cleaning of data, it was divided into two sets, the training and testing set.

The training set underwent the forecasting of predictors using Seasonal Autoregressive Moving Average (SARIMA) which was discussed in Section 2. The same set of data was used to develop a Hidden Markov Model by calculating the Indicators, Average True Time Range and Logarithmic Returns respectively. The data frame for Hidden Markov Model was performed using the generated Average True Range and Logarithmic Returns. The generation of Hidden Markov Model took place using the Gaussian response distribution and the data frame and the model was fitted into the data sets. The Viterbi Path was obtained using the model and utilized the Baum Welch Algorithm to obtain the final matrix.

The y-intercept of the model was obtained by getting the multiplicative inverse of the trace of the matrix. The intercept and the weights obtained from the neural networks as discussed in Section III was used to create the enhanced model as shown in (12). The specific model for both Apple and Nokia Data was presented in (13) and (14).

\[
\hat{y} = \frac{1}{2}\left(\frac{1}{2}|a_1 x_1 + a_2 x_2 + a_3 x_3 + a_4 \log \log (x_4) | + b + c \right) + y_{NN}
\]  

(12)

where

- \(\hat{y}\) = predicted close price
- \(a\) = weights of the predictors in Neural Networks
- \(x\) = forecasted values of predictors from ARIMA
- \(b\) = y-intercept of the model as computed using the Baum-Welch Algorithm
- \(c\) = intercept to Neural Networks model
- \(y_{NN}\) = close price intercept in Neural Networks model
Using the model in (13) and (14), the results of the prediction of close price for two weeks for both Apple and Nokia data was shown in Table 4.

\[
\hat{y}_{\text{APPLE}} = \frac{1}{2} \left( \frac{1}{2} \left( 10.52866x_1 - 0.52866x_2 - 1.89025x_3 - 0.17921 \log(x_4) \right) + 2.947413 + 0.08061 \right) + 140.81839 
\]  
\[
\hat{y}_{\text{NOKIA}} = \frac{1}{2} \left( \frac{1}{2} \left( 1.9046x_1 + 0.32655x_2 + 0.19645x_3 + 0.009572 \log(x_4) \right) + 2.947413 - 0.00103 \right) + 3.44605 
\]

Using the model in (13) and (14), the results of the prediction of close price for two weeks for both Apple and Nokia data was shown in Table 4.

**Table 4: The Predicted Close Price of Apple and Nokia Using Enhanced Hidden Markov Model**

<table>
<thead>
<tr>
<th>Date</th>
<th>Apple Close Price</th>
<th>Nokia Close Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>2018-12-31</td>
<td>144.061</td>
<td>5.95149</td>
</tr>
<tr>
<td>2019-01-01</td>
<td>143.9235</td>
<td>5.948356</td>
</tr>
<tr>
<td>2019-01-02</td>
<td>143.9175</td>
<td>5.951159</td>
</tr>
<tr>
<td>2019-01-03</td>
<td>144.0198</td>
<td>5.949902</td>
</tr>
<tr>
<td>2019-01-04</td>
<td>144.0434</td>
<td>5.949633</td>
</tr>
<tr>
<td>2019-01-07</td>
<td>143.876</td>
<td>5.944049</td>
</tr>
<tr>
<td>2019-01-08</td>
<td>143.8893</td>
<td>5.941604</td>
</tr>
<tr>
<td>2019-01-09</td>
<td>143.9569</td>
<td>5.944384</td>
</tr>
<tr>
<td>2019-01-10</td>
<td>143.9775</td>
<td>5.94312</td>
</tr>
<tr>
<td>2019-01-11</td>
<td>143.9547</td>
<td>5.942807</td>
</tr>
</tbody>
</table>
6 Model Evaluation
To evaluate the performance of the Enhanced Hidden Markov Model the error and accuracy of the model were computed using MAPE whose formula was shown in (11). To verify if there is no significant difference between the actual and predicted close price, the paired t-test was employed since both sets of data were normally distributed. The normality was performed using the Shapiro-Wilk Test which obtained a p-value of 0.7639 and 0.6636 for actual and predicted closed price of Apple. The Nokia data obtained a p-value of 0.587 and 0.1258 for actual and predicted close price respectively.

6.1 Error and Accuracy
As presented in Section 5, the predicted close price of Apple and Nokia was shown in Table 4, while Table 5 shows the actual close price for the two data sets.

Table 5: The Actual Close Price of Apple and Nokia Using Enhanced Hidden Markov Model

<table>
<thead>
<tr>
<th>Date</th>
<th>Apple Close Price</th>
<th>Nokia Close Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>2018-12-31</td>
<td>157.74</td>
<td>5.82</td>
</tr>
<tr>
<td>2019-01-01</td>
<td>149.4567</td>
<td>5.765</td>
</tr>
<tr>
<td>2019-01-02</td>
<td>157.92</td>
<td>5.74</td>
</tr>
<tr>
<td>2019-01-03</td>
<td>142.19</td>
<td>5.57</td>
</tr>
<tr>
<td>2019-01-04</td>
<td>148.26</td>
<td>5.93</td>
</tr>
<tr>
<td>2019-01-07</td>
<td>147.93</td>
<td>6.02</td>
</tr>
<tr>
<td>2019-01-08</td>
<td>150.75</td>
<td>6.15</td>
</tr>
<tr>
<td>2019-01-09</td>
<td>153.31</td>
<td>6.21</td>
</tr>
<tr>
<td>2019-01-10</td>
<td>153.8</td>
<td>6.14</td>
</tr>
<tr>
<td>2019-01-11</td>
<td>152.29</td>
<td>6.08</td>
</tr>
</tbody>
</table>

Table 6 shows the error and accuracy of predictions of Enhanced Hidden Markov Model for Apple and Nokia data.

Table 6. Error and Accuracy of the Predicted Close Price Using Apple Data

<table>
<thead>
<tr>
<th>DATA SET</th>
<th>ERROR (%)</th>
<th>ACCURACY (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Apple</td>
<td>5.0624</td>
<td>94.9376</td>
</tr>
<tr>
<td>Nokia</td>
<td>3.0660</td>
<td>96.9340</td>
</tr>
</tbody>
</table>

6.2 Testing for Significant Difference
At 5% level of significance, it was hypothesized that there is no significant difference between the predicted and actual price. Using the paired t-test whose formula shown in (15). The result of testing was presented in Table 7.

\[ t = \frac{d - d_0}{s_d \sqrt{n}} \]  

where

- \( t \) = the computed value of t;
- \( d \) = average of the deviation of the results between predicted and actual stock close price;
- \( d_0 \) = assumed to be 0 since its Ho claims that there is no significant difference between the predicted and actual price;
- \( n \) = total number of test cases;
- \( s_d \) = the standard deviation of the difference between the predicted and actual close price.
7 Conclusions
This paper used Seasonal Autoregressive Integrated Moving Average to forecast the predictors of close price. The forecasting models developed in this paper produced good metrics for accuracy which only implies that SARIMA is useful in dealing with the stocks data. The results of SARIMA were utilized in the Enhanced Hidden Markov Model to predict the close price of stock. Mean Absolute Percentage Error was used to measure the performance of the developed model and the outcome obtained a promising result for prediction with at least 95% accuracy. In order to verify if there is no difference between the actual and predicted close price, a paired t-test was implemented. Nokia Data concluded that at 5% level of significance, there is no significant difference between the predicted and actual close price. However, since there is a significant difference in the actual and predicted close price in the Apple Data, it is recommended to further explore the inputs of the Enhanced Hidden Markov Model. Future researchers are encouraged to use the developed model in other fields of applications.

8 Declarations

8.1 Competing Interests
Authors declare that no potential conflict of interest exists related to this article.

8.2 Acknowledgement
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